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INVESTIGATION OF THE CHARACTERISTICS OF CONJUGATE HEAT AND MASS TRANSFER IN SPATIAL FLOW PAST A SPHERE-BLUNTED CONE AND BLOWING-IN OF A GAS FROM THE SURFACE OF BLUNTNESS

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Some means of controlling thermal regimes in the case of a high-enthalpy flow past a blunt body at an angle of attack with a simultaneous influence of the gas blown-in from the permeable bluntness surface and overflowing of heat in the shell of the material are considered. The effectiveness of applying highly conducting materials in order to lower maximum temperatures in a zone of a screen is shown.

Introduction. Among the most complex problems in designing high-velocity flying vehicles is the problem of thermal shielding of a structure. Higher demands made upon the accuracy of determining the characteristics of heat and mass transfer in the shell of a body immersed in a flow necessitate the solution of the problem in a joint statement, which is confirmed by the results of investigations [1–3].

An analysis of the characteristics of conjugate heat and mass transfer with a laminar mode of flow in a boundary layer [3] has shown that a reliable means of protecting the structure from overheating is blowing-in of a gas-cooler. This weakens the heat flux supplied to the surface, and heat is removed during filtration of the gas in the pores. In [3, 4], heat-conducting materials that ensure a decrease in the surface temperature of the body in the zone of the screen are studied. In this work, we present results of solution of the problem of heating through the shell of a sphere-blunted cone past which a supersonic air stream flows at an angle of attack, with allowance for different flow regimes in a boundary layer. To reduce the maximum surface temperature, highly conducting materials and blowing-in of a gas-cooler from a surface of porous bluntness were used. The characteristics of conjugate heat and mass transfer are sought from the solution of a system of equations which describe a change in averaged values in the boundary layer, energy-conservation equations for porous spherical bluntness, and the nonstationary heat-conduction equation for the conical portion of the shell.

Statement of the Problem. In [5], estimations of the time of relaxation in the gas and condensed phases were made. Based on these estimations, for a model of a perfect gas the system of equations of a three-dimensional boundary layer in a natural coordinate system fixed on the body in a flow has the form

$$\frac{\partial (\rho u r_{\mathrm{w}})}{\partial s} + \frac{\partial (\rho v r_{\mathrm{w}})}{\partial n} + \frac{\partial (\rho w)}{\partial \eta} = 0\,,$$

$$\rho\left(u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial n} + \frac{w}{r_{\mathrm{w}}}\frac{\partial u}{\partial \eta} - \frac{w^2}{r_{\mathrm{w}}}\frac{\partial r_{\mathrm{w}}}{\partial s}\right) = -\frac{\partial P_{\mathrm{e}}}{\partial s} + \frac{\partial}{\partial n}\left(\mu_\Sigma \frac{\partial u}{\partial n}\right),$$

$$\rho\left(u\frac{\partial w}{\partial s} + v\frac{\partial w}{\partial n} + \frac{w}{r_{\mathrm{w}}}\frac{\partial w}{\partial \eta} + \frac{uw}{r_{\mathrm{w}}}\frac{\partial r_{\mathrm{w}}}{\partial s}\right) = -\frac{1}{r_{\mathrm{w}}}\frac{\partial P_{\mathrm{e}}}{\partial \eta} + \frac{\partial}{\partial n}\left(\mu_\Sigma \frac{\partial w}{\partial n}\right),\tag{1}$$

$$\rho\left(u\frac{\partial h}{\partial s} + v\frac{\partial h}{\partial n} + \frac{w}{r_{\mathrm{w}}}\frac{\partial h}{\partial \eta}\right) = \frac{\partial}{\partial n}\left(\frac{\mu_\Sigma}{\mathrm{Pr}_\Sigma}\frac{\partial h}{\partial n}\right) + u\frac{\partial P_{\mathrm{e}}}{\partial s} + \frac{w}{r_{\mathrm{w}}}\frac{\partial P_{\mathrm{e}}}{\partial \eta} + \mu_\Sigma\left[\left(\frac{\partial u}{\partial n}\right)^2 + \left(\frac{\partial w}{\partial n}\right)^2\right],$$

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$$P_{\rm e} = \rho h \frac{\gamma - 1}{\gamma}\,, \quad P_{\rm e} = P_{\rm e}\,(s, \eta)\,.$$

For a porous spherical shell ($0 \le s \le s_1$), under the assumption that the process of filtration of a blown-in gas normal to the surface in the considered coordinate system fixed at the symmetry axis of the body is one-dimensional, we write the following energy-conservation equation [4]:

$$(\rho c_p)_1\,(1-\phi)\frac{\partial T_1}{\partial t} = \frac{1}{r_1 H_1}\left\{\frac{\partial}{\partial n_1}\left[r_1 H_1 \lambda_1\,(1-\phi)\frac{\partial T_1}{\partial n_1}\right] + \frac{\partial}{\partial s}\left[\frac{r_1 \lambda_1}{H_1}\,(1-\phi)\frac{\partial T_1}{\partial s}\right] + \right.$$

$$\left. + \frac{\partial}{\partial \eta}\left[\frac{H_1 \lambda_1}{r_1}\,(1-\phi)\frac{\partial T_1}{\partial \eta}\right]\right\} + (\rho v)_{\rm w}\,\frac{r_{1\rm w}}{r_1 H_1}\,c_{pg}\,\frac{\partial T_1}{\partial n_1}\,, \tag{2}$$

where $0 \le n_1 \le L$; $0 \le \eta \le \pi$; $H_1 = (R_N - n_1)/R_N$; $r_1 = (R_N - n_1)\sin(\bar{s})$, and $\bar{s} = s/R_N$.

For the conical part of the body ($s_1 \le s \le s_{\rm per}$) the heat-conduction equation takes the form [4]

$$r_2\,(\rho c_p)_2\,\frac{\partial T_2}{\partial t} = \frac{\partial}{\partial n_1}\left(r_2 \lambda_2 \frac{\partial T_2}{\partial n_1}\right) + \frac{\partial}{\partial s}\left(r_2 \lambda_2 \frac{\partial T_2}{\partial s}\right) + \frac{1}{r_2}\frac{\partial}{\partial \eta}\left(\lambda_2 \frac{\partial T_2}{\partial \eta}\right), \tag{3}$$

where $r_2 = (R_N - n_1)\cos\theta + (s - s_1)\sin\theta$.

The boundary and initial conditions for system (1)–(3) are: on the outer edge of the boundary layer for $n \to \infty$

$$u \to u_{\rm e}\,(s, \eta)\,, \quad w \to w_{\rm e}\,(s, \eta)\,, \quad h \to h_{\rm e}\,(s, \eta) \tag{4}$$

($u_{\rm e}$, $w_{\rm e}$, and $h_{\rm e}$ in (4) and $P_{\rm e}$ in (1) were determined by solving the system of Euler equations by the method suggested in [6]); on the surface of the body immersed in a flow at $n = 0$ [3, 4]

$$u\,(s, \eta) = w\,(s, \eta) = 0\,; \quad (\rho v) = (\rho v)_{\rm w}\,(s, \eta)\,, \quad 0 \le s < s_1\,;$$

$$\rho v = 0\,, \quad s_1 \le s \le s_{\rm per}\,. \tag{5}$$

On the outer surface of the body immersed in a flow with $0 \le \eta \le \pi$ the balance relations are written as

$$\left.\left(\frac{\mu}{\rm Pr}\frac{\partial h}{\partial n}\right)\right|_{\rm w} - \varepsilon_1 \sigma T_{1\rm w}^4 = -\,\lambda_1\,(1-\phi)\left.\frac{\partial T_1}{\partial n_1}\right|_{\rm w}\,,$$

$$\left.\left(\frac{\mu}{\rm Pr}\frac{\partial h}{\partial n}\right)\right|_{\rm w} - \varepsilon_2 \sigma T_{2\rm w}^4 = -\,\lambda_2 \left.\frac{\partial T_2}{\partial n_1}\right|_{\rm w}\,, \tag{6}$$

whereas on the inner surface of the hemisphere and on the conical part they are represented as [4]

$$\lambda_1\,(1-\phi)\left.\frac{\partial T_1}{\partial n_1}\right|_{n_1 = L} = -\,\frac{r_{1\rm w} c_{pg}\,(\rho v)_{\rm w}}{(r_1 H_1)_{n_1 = L}}\,(T_{1L} - T_{\rm in})\,, \quad 0 \le s < s_1\,;$$

$$\lambda_2 \left.\frac{\partial T_2}{\partial n_1}\right|_{n_1 = L} = 0\,, \quad s_1 \le s \le s_{\rm per}\,. \tag{7}$$

On the sphere–cone conjugation ring at $s = s_1$ the conditions of an ideal contact are placed and at the extreme section of the body at $s = s_{\rm per}$ the following adiabatic condition is adopted:

$$\lambda_1 (1-\phi)\frac{1}{H_1}\frac{\partial T_1}{\partial s}\bigg|_{s=s_1-0} = \lambda_2 \frac{\partial T_2}{\partial s}\bigg|_{s=s_1+0}, \quad T_1\Big|_{s=s_1-0} = T_2\Big|_{s=s_1+0},$$

$$\frac{\partial T_2}{\partial s}\bigg|_{s=s_{\text{per}}} = 0 . \tag{8}$$

In the presence of the plane of flow symmetry

$$\frac{\partial T_i}{\partial \eta}\bigg|_{\eta=0} = \frac{\partial T_i}{\partial \eta}\bigg|_{\eta=\pi} = 0 , \quad i = 1, 2 . \tag{9}$$

The initial conditions for system (1)–(3) are

$$T_1\Big|_{t=0} = T_2\Big|_{t=0} = T_{\text{in}} . \tag{10}$$

Equations (1) were written on the assumption of invariable gas composition; it is valid when the composition of the blown-in gas coincides with that of the incoming flow. Air is used as a blown-in gas.

To describe a turbulent flow a model of a turbulent boundary layer described in detail in [1] was used. For a laminar, transient, and developed turbulent regimes of a flow in a boundary layer the following relations are valid:

$$\mu_\Sigma = \mu + G\mu_{\text{t}} , \quad \text{Pr}_\Sigma = \frac{\mu_\Sigma \,\text{Pr}\,\text{Pr}_{\text{t}}}{\mu \text{Pr}_{\text{t}} + G\mu_{\text{t}}\text{Pr}} .$$

We will consider a three-layer algebraic model of turbulence provided that in a laminar viscous sublayer near the surface of a body in a flow the condition $\mu > \mu_{\text{t}}$ is continuously fulfilled. The inner region of the turbulent core is described by the Van Driest–Cebeci formula [7], which for a three-dimensional flow has the form

$$\mu_{\text{t,i}} = 0.16\rho n^2 \left[1 - \exp\left(-\frac{n^+}{A^+}\right)\right]^2 \left[\left(\frac{\partial u}{\partial n}\right)^2 + \left(\frac{\partial w}{\partial n}\right)^2\right]^{0.5} ,$$

$$n^+ = \frac{n\rho}{\mu}\left(\frac{\tau_{\text{w}}}{\rho}\right)^{0.5} ,$$

where in the presence of blowing

$$A^+ = 26\left(V_{\text{w}}^+\right)^{0.5}\left[\frac{\mu}{\mu_{\text{e}}}\left(\frac{\rho_{\text{e}}}{\rho_{\text{w}}}\right)^2 P^+ (1-\psi) + V_{\text{w}}^+\psi\right]^{-0.5} ,$$

$$P^+ = -\frac{\mu_{\text{e}}}{\rho_{\text{e}}^2 (\tau_{\text{w}}/\rho_{\text{w}})^{0.5}\, U_{\text{e}}}\left(u_{\text{e}}\frac{\partial P_{\text{e}}}{\partial s} + \frac{w_{\text{e}}}{r_{\text{w}}}\frac{\partial P_{\text{e}}}{\partial \eta}\right),$$

$$W_{\text{w}}^+ = \frac{(\rho v)_{\text{w}}}{\rho_{\text{w}} (\tau_{\text{w}}/\rho_{\text{w}})^{0.5}} , \quad U_{\text{e}}^2 = u_{\text{e}}^2 + w_{\text{e}}^2 ,$$

$$\psi = \exp\left(11.8\,\frac{\mu_{\text{w}}}{\mu}\,V_{\text{w}}^+\right), \quad \tau_{\text{w}} = \mu_{\text{w}}\left[\left(\frac{\partial u}{\partial n}\bigg|_{\text{w}}\right)^2 + \left(\frac{\partial w}{\partial n}\bigg|_{\text{w}}\right)^2\right]^{0.5} ,$$

and in its absence

$$A^+ = 26\left[1 - 11.8\,\frac{\mu_w}{\mu_e}\left(\frac{\rho_e}{\rho_w}\right)^2 P^+\right]^{-0.5}.$$

The outer part of the turbulent core is described by Spalding's formula [8]

$$\mu_{t,e} = \rho\,(0.089 n_e)^2\left[\left(\frac{\partial u}{\partial n}\right)^2 + \left(\frac{\partial w}{\partial n}\right)^2\right]^{0.5}.$$

Transition from a laminar model of flow to a developed turbulent one on the spherical bluntness is described by the Dhawan–Narasimha formula [9]:

$$G = 1 - \exp\left(-3.507\,\frac{s - s_b}{s_a - s_b}\right),$$

where the values of the coordinates s_b and s_a were selected from the conditions of the correspondence of the results obtained to the data of experimental work [10]. Transition from the inner region of the turbulent core to the outer one occurs when the condition $\mu_{t,i} = \mu_{t,e}$ is satisfied.

It was assumed that the flows considered had conditions identical with those of work [10]: the same incoming flow Mach numbers, initial temperatures of the body, and geometries of the models. In calculating the state of the temperature field of the models, the point of stability loss was prescribed on the basis of experimental data of [10].

Methods of Calculation and Initial Data. The boundary-value problem (1)–(10) was solved numerically [11, 12] in Dorodnitsyn's variables. For the boundary-layer equations, using the method suggested in [11], physically adapted numerical difference schemes were obtained that ensure the joining of the sought-for characteristics at the interface between the laminar sublayer and turbulent core and taking into account the character of the change in μ_t across the boundary layer. This allows one to perform calculations in wide ranges of Reynolds number and intensities of the blown-in gas flow rate. The solution of the three-dimensional nonstationary equations (2) and (3) was performed on the basis of the locally one-dimensional scheme of splitting [12].

The geometry of the model, the flow parameters, the intensity and the law of the rate of the gas flow blown-in from the surface of the spherical bluntness, and the porosity of the material of bluntness were assumed as follows: $\mathrm{Pr} = 0.72$, $\mathrm{Pr_t} = 1$, $\mathrm{Re} = 3.8\cdot10^6$, $\mathrm{M}_\infty = 5$, $R_N = 0.0508$ m, $P_{e0} = 10^6$ N/m^2, $(\rho v)_w$ = const, $T_{in} = 288$ K, $T_{e0} = 1500$ K, $h_{e0} = 1.614\cdot10^6$ J/kg, $\gamma = 1.4$, $\varepsilon_1 = \varepsilon_2 = 0.85$, $L = 2.2\cdot10^{-3}$ m, $\theta = 5^o$, $\beta = 10^o$, and $\varphi = 0.34$. The thermophysical characteristics of the materials used correspond to asbestos cement ($\lambda = 0.349$ W/(m·K), $c_p = 837$ J/(kg·K), $\rho = 1800$ kg/m^3) and to copper ($\lambda = 386$ W/(m·K), $c_p = 370$ J/(kg·K), $\rho = 8950$ kg/m^3) [13], as well as to the limiting case $\lambda \to \infty$.

For $\beta/\theta = 2$, the calculation of the conical part of the body terminates at $s_{per}/R_N = 5$. As follows from [14], separation of the boundary layer on the leeward side may occur at $s/R_N \geq 5.25$ for $\beta/\theta > 2$. The calculations were performed by the marching method despite the nonmonotonic behavior of pressure in the circumferential direction η. The results were obtained in a wide range of the blowing parameters, including the regime of displacement of the boundary layer in the vicinity of the forward stagnation point. In this case, the regime of the overflowing of heat from the conical peripheral part to the forward spherical part sets in. The bluntness becomes a heat sink, and simultaneously a decrease in the temperature of the shell due to heat transfer in the course of the filtration of a cold blown-in gas in the pores of the spherical shell occurs.

Analysis of the Results of Numerical Solution. Figure 1a presents the dependences of the convective heat flux from the gas phase q_w^0 and Fig. 1b — the surface temperatures T_w on the longitudinal coordinate s/R_N on the windward and leeward sides of the plane of symmetry of the body shell made from copper at $(\rho v)_w = 0$ (curves 1 and 2 for q_w^0 and T_w at $t = 0$ coincide). In order to estimate the circumferential overflowing of heat η, a two-dimensional problem in a condensed phase was solved which resulted from Eqs. (2) and (3) in the absence of circumferential overflowing of heat (curve 2 in Fig. 1b). It is the distribution of the radiative equilibrium temperature $T_{w,eq}$ in the plane

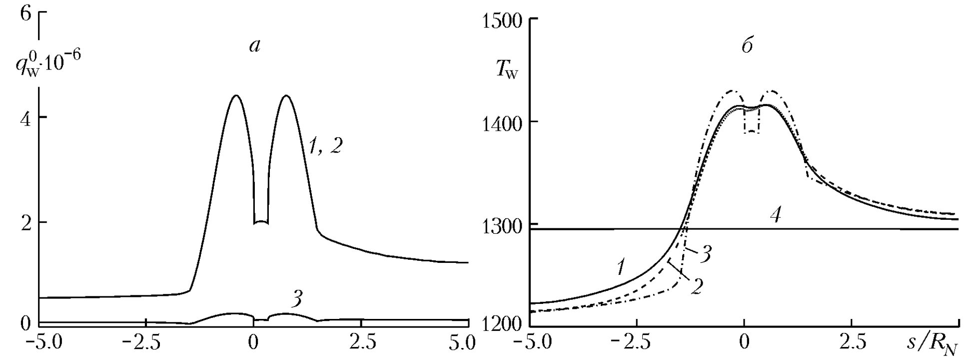

Fig. 1. Convective heat flux from a gas phase q_w^0 (a) [1, 2) t = 0; 3) 200 sec (steady-state $t \to \infty$ mode of body heating)] and surface temperature T_w (b) [1) a three-dimensional process of heat and mass transfer in a condensed phase; 2) two-dimensional; 3) one-dimensional; 4) $\lambda_i \to \infty$; t = 200 sec] vs. the longitudinal coordinate in the plane of symmetry of flow on the windward and leeward sides. q_w^0, W/m^2; T_w, K.

of symmetry on the windward and leeward sides which is determined from the condition of energy conservation on the spherical and conical surfaces:

$$q_w + c_{pg}\,(\rho v)_w\,(T_{in} - T_{w,eq}) = \varepsilon_1 \sigma T_{w,eq}^4\,, \quad q_w = \varepsilon_2 \sigma T_{w,eq}^4 \tag{11}$$

and which determines the maximum possible temperature of the surface in the absence of heat overflowing in the longitudinal and circumferential directions, as shown in Fig. 1b (curve 3).

In Fig. 1a, a characteristic feature is profile 1 which corresponds to the initial instant of time. The right part of this curve starts with a "shelf," which corresponds to the laminar portion of flow with a maximum in the vicinity of the stagnation point which is spaced-out from the ordinate axis by the value of the angle of attack. The heat flux decreases before the beginning of the transition section with the coordinate of the point of the loss of stability approximately equal to 10^o in the natural coordinate system fixed at the stagnation point. An increase in the degree of flow turbulence downstream of the transition point is accompanied by an increase in the heat flux to the surface of the body in a flow. The heat-flux maximum corresponds to the region of the acoustic line. Downstream of the acoustic line further acceleration of the flow and its expansion occur, which are accompanied by a decrease in the heat content. This, in turn, leads to a decrease in the heat flux to the body. When the heat flux passes to the conical part, its distribution can be nonmonotonic in the directions longitudinal and normal to the body because of the nonmonotonic behavior of pressure.

As a result of the heating from outside, the temperature of the solid-body surface increases and its highest value corresponds to the value of $T_{w,eq}$ in the region of maximum heat flux for a turbulent mode of flow in the boundary layer near the stagnation point. Allowance for the heat overflowing yields a decreasing T_w for a high-conducting material of the type of copper and, with the circumferential overflowing of heat not being taken into account, reduces the surface temperature on the leeward side by 15 K and raises it on the windward side at t = 200 sec (curves 1 and 2, in Fig. 1b). When a steady-state regime is attained, the progress of the process T_w on the leeward side exceeds the radial equilibrium temperature $T_{w,eq}$ due to the longitudinal and circumferential overflowing of heat.

Apart from the calculation of $T_{w,eq}$ from conditions (11), we considered the solution of the given problem for a material with low thermal conductivity, of the type of asbestos cement. For such a material, as soon as a steady-state regime is attained, the surface temperature coincides with the values of $T_{w,eq}$, since for asbestos cement the process of heating is one-dimensional. When $\lambda_i \to \infty$, i = 1, 2, there is a considerable decrease in T_w (straight line 4 in Fig. 1b) of the spherical portion of the shell and equalization of the temperature profile of the body in a flow, whereas the values of the surface temperatures agree with the results of calculations by the formulas given in [4].

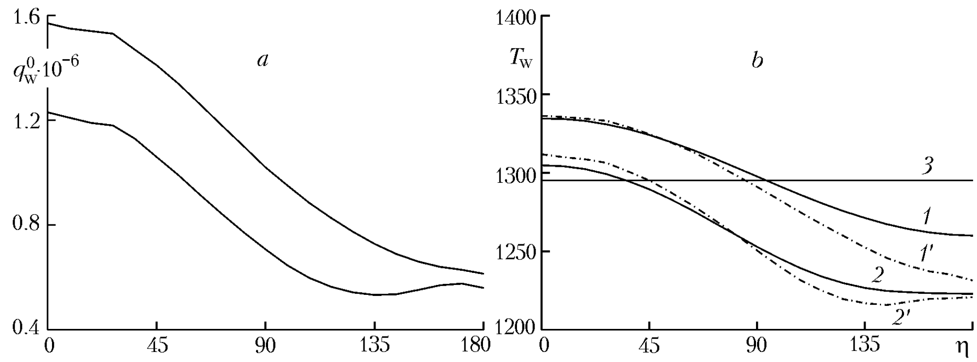

Fig. 2. Distribution of the heat flux q_w^0 (a) [1) $\bar{s} = 2$; 2) $\bar{s} = \bar{s}_{per}$] and of the surface temperature T_w (b) [1, 2) three-dimensional process of heat and mass transfer in a body; 1′, 2′) one-dimensional; 1, 1′) $\bar{s} = 2$; 2, 2′) $\bar{s} = \bar{s}_{per}$; 3) $\lambda_i \to \infty$, $i = 1, 2$] along the circumferential coordinate η on the conical part of the body at $t = 200$ sec for $(\rho v)_w = 0$. q_w^0, W/m^2; T_w, K.

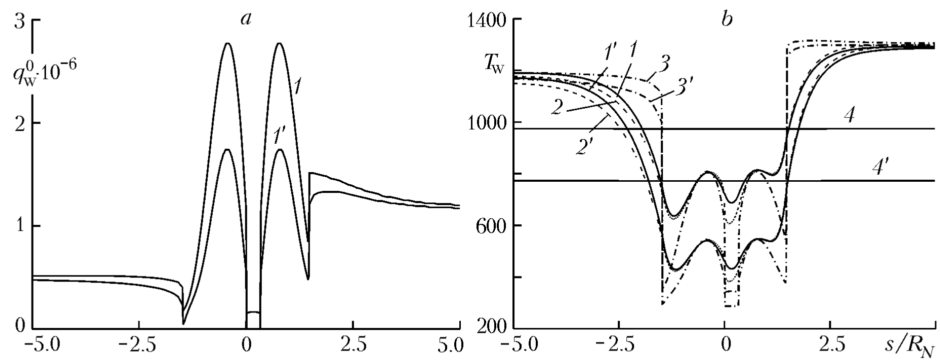

Fig. 3. Convective heat flux q_w (a) and surface temperature T_w (b) [1) a three-dimensional process of heat and mass transfer in a condensed phase; 2) two-dimensional; 3) one-dimensional; 4) $\lambda_i \to \infty$, $i = 1, 2$] vs. the longitudinal coordinate in the plane of symmetry of the flow: 1–4) $(\rho v)_w = 3$ kg/(m^2·sec); 1′–4′) 6. q_w^0, W/m^2; T_w, K.

Figure 2 presents the values of q_w^0 and T_w along the circumferential coordinate η on the conical portion of the body in the section $\bar{s} = 2$, which is close to the plane of junction of the sphere and cone and $\bar{s} = \bar{s}_{per}$ (peripheral region). In the section with $\bar{s} = \bar{s}_{per}$, the difference between the radiative equilibrium temperature and the surface temperature is observed in the most thermally stressed section with $\eta = 130^o$ (~10 K) at $t = 200$ sec, whereas the maximum difference between T_w and $T_{w,eq}$ at $t = 200$ sec is noted on the portion of the shell for $\bar{s} = 2$, $\eta = 180^o$ and amounts to about 35 K (Fig. 2b).

We will consider the influence of the gas-coolant rate of flow from the surface of bluntness. Figure 3 presents the dependences of the convective heat flux from the gas phase q_w and of the surface temperature T_w for the initial instant of time $t = 0$ (a) and in the case of a steady-state mode of the occurrence of the process ($t = 200$ sec) (b). It is seen that at $t = 0$ the blowing-in of the gas-coolant from the porous spherical bluntness leads to a decrease in the maximum of q_w on the sphere by a factor of 1.6–2.5 and to its decrease on the conical part of the body (curves 1, 2 in Fig. 1a and 1, 1′ in Fig. 3a). Moreover, here heat is absorbed during gas filtration in the pores of spherical bluntness. As a result, the temperature of the permeable bluntness in the most thermally stressed section $\eta = 0$ for $(\rho v)_w$ = 3 and 6 kg/(m^2·sec) and at $t = 200$ sec does not exceed 850 and 550 K, respectively.

Figure 4 demonstrates the distribution of the surface temperature over the circumferential coordinate in the case of a steady-state mode of the process of body heating ($t = 200$ sec). As follows from Figs. 3 and 4, the distribution of the surface temperature on injection of the gas-coolant differs qualitatively from the distribution of T_w at

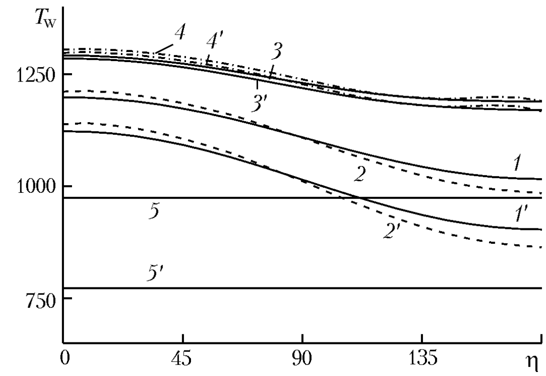

Fig. 4. Distribution of the surface temperature along the circumferential coordinate on the conical part of the body: 1, 3) a three-dimensional process of heat and mass transfer in a body: 2) two-dimensional; 4) one-dimensional; 1, 2) $\bar{s} = 2$; 3, 4) $\bar{s} = \bar{s}_{per}$; 5) $\lambda_i \to \infty$, $i = 1, 2$; 1–5) $(\rho v)_w = 3$ kg/(m^2·sec); 1′–5′) 6. T_w, K.

$(\rho v)_w = 0$ for different finite values of λ_i. On the porous portion of the body T_w may exceed the corresponding value of $T_{w,eq}$, whereas on the conical portion, both on the windward and leeward sides, the steady-state value of T_w becomes lower than $T_{w,eq}$ due to heat sink into the porous bluntness. The surface temperature corresponding to $\lambda_i \to \infty$, $i = 1, 2$, and to the steady-state three-dimensional process of heat transfer at $(\rho v)_w \neq 0$ (Fig. 3b, straight lines 4, 4′; Fig. 4, straight lines 5, 5′) becomes twice as low in comparison with the data presented in Figs. 1b and 2b. The results obtained at $(\rho v)_w = 0$–6 kg/(m^2·sec) confirm the conclusions on the advisability of using high-conducting materials that ensure an intense sink of heat into the region of permeable bluntness; moreover, it follows from Figs. 3b and 4 that injection noticeably decreases maximum temperatures, but the use of heat-conducting materials gives a much greater decrease in the maximum temperature of the shell of the conical surface in the screen zone. It should be noted here that because of the weakening of the effect of injection on the characteristics of heat and mass transfer in the gas phase in a turbulent mode of flow in the boundary layer, the effectiveness of using highly conducting materials is somewhat lower than in laminar flow in a boundary layer [3] in agreement with the results of [15].

In addition to the solution of the problem in a joint statement, we also studied the problem of the legitimacy of using separate statements for the case of a given coefficient of heat transfer from the side of the gas phase for an isothermal (at $t = 0$) body surface. At $(\rho v)_w = 0$, in contrast to the laminar mode of flow in a boundary layer [3], where a separate statement can be used for calculating the temperature field, the use of the coefficient of heat transfer to an isothermal surface in a turbulent mode of flow in a boundary layer leads to an increase in T_w up to 5% for a copper shell on the conical part, whereas at the value of the heat-transfer coefficient obtained by the formulas of [16] it leads to a decrease in T_w by 4% at $t = 200$ sec, as compared to the solution of the problem in a joint statement.

In the presence of injection $0 < (\rho v)_w \le 6$ kg/(m^2·sec) in both laminar and turbulent modes of flow in a boundary layer, the application of an approximate approach based on the use of the coefficient of heat transfer to an isothermal surface of a copper shell leads to an appreciable divergence (up to 11–20%) of the surface temperature T_w in the screen zone in comparison with the exact solution of the problem of heating in a joint statement. In the general case of a nonisothermal surface, this is due to the complex structure of the heat-transfer coefficient, which, as is shown in [1], includes a term containing a local derivative related to the temperature drop $\partial T_w/\partial\bar{s}/(T_{e0} - T_w)$, which becomes appreciable in the zone of the thermal screen where significant temperature gradients $\partial T_w/\partial\bar{s}$ are realized.

CONCLUSIONS

1. A mathematical model for calculating a three-dimensional conjugate heat and mass transfer in three-dimensional flow past a sphere-blunted cone with allowance for various regimes of flow in a boundary layer has been developed.

2. To reduce a maximum temperature of the shell in the zone of a screen it is advisable to use highly conducting materials in combination with gas-coolant injection through porous bluntness.

3. The effectiveness of using highly conducting materials in a turbulent mode of flow in a boundary layer is somewhat lower than in laminar mode of flow in a boundary layer.

4. The joint statement of the problem makes it possible to take into account the influence of the nonisothermicity of the shell wall on the characteristics of heat and mass transfer in a boundary layer.

NOTATION

c_p, heat capacity, J/(kg·K); G, coefficient of longitudinal intermittence; h, static enthalpy, J/kg; L, thickness of the shell of a body in a flow, m; M_∞, Pr, Re, Mach, Prandtl, and Reynolds numbers of an incoming flow; n, coordinate along the normal to the body surface directed to the gas phase; n_1, coordinate along the normal to the body surface directed to the condensed phase; P, pressure, N/m^2; q_w, convective heat flux from a gas phase, W/m^2; r_w, r_1, H_1, Lamé coefficients; R_N, radius of spherical bluntness, m; s, longitudinal coordinate, m; s_b and s_a, coordinates of the beginning and end of the transition zone m; t, time, sec; T, temperature, K; u, v, w, components of the mean-mass velocity vector, m/sec, in a natural coordinate system s, n, η; β, angle of attack, deg; ε_i, $i = 1, 2$, emissivities of the body surface; θ, conicity angle, deg; λ, thermal conductivity, W/(m·K); η, circumferential coordinate, deg; μ, coefficient of dynamic viscosity, kg/(m·sec); ρ, density, kg/m^3; $(\rho v)_w$, gas-coolant flow rate, kg/(m^2·sec); σ, Stefan–Boltzmann constant, W/(m^2·K^4); Φ, porosity. Subscripts: e, outer edge of the boundary layer; e0, flow stagnation point; w, surface of the body in a flow; g, gas phase of a porous spherical shell; i, inner region; eq, equilibrium; t, turbulent mode of flow in a boundary layer; Σ, total value; 1, 2, characteristics of the condensed phase of the spherical and conical parts of the body; in, initial conditions; per, peripheral portion of the shell. Superscripts: 0, heat flux from a gas phase in the absence of injection; bar, dimensionless quantity.

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